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## The Analysis of Pile's Bearing Capability for a Pile Field of Short Grid Pitch in Hydrocompactive Soil of Type II: Theoretical Basics of Method

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**Abstract.** The deformation hypotheses about the work of hydrocompactive soil at the space between piles have been suggested and based, and on their basis the equilibrium equation of watered hydrocompactive strata loaded with pile foundation has been obtained. The main assumptions in development of this equation are as follows: 1) In the middle of the row's adjacent piles the soil vertical displacement after foundation settlement but before watering is linear function of depth; 2) The vertical deformation variation caused by watering is defined only by the relative hydrocompaction subsidence; 3) The tangent surface forces applied from a soil to a pile are established for linear-elastic soil with account of the friction force upper-limits. The obtained equation for the hydrocompactive strata's state contains unknown parameter which is the minimum settlement of no-watered soil within the boundaries of a pile row. To determine the latter, the finite element simulation technique is grounded. The algorithm for solving the equation for hydrocompactive strata's state has been elaborated to derive the safe design load on a pile in hydrocompactive soil of type II. The results of pile design load analysis, which is based on this solver, is presented and comparison with standard analysis technique is given. It is established that the safe bearing load of a pile is higher than one being calculated by actual standards, that is caused by relief of hydrocompactive strata due to forces counteracting against negative friction.

In analysis of the safe bearing load on a pile, due to SP<sup>1</sup> 24.13330.2011 "SNIP<sup>2</sup> 2.02.03-85 Pile Foundations" there is no difference in work of single pile and a pile in a pile field (PF). Nevertheless, in the case of hydrocompactive soil of type II and a PF of short enough grid pitch, a significant difference in altitude distribution of vertical pressure is possible in watered collapsible soil between piles of field, relative to the same soil with a single pile. In the upper part of hydrocompactive stratum (HS) the soil being collapsed hangs on the piles after wa-

tering, and the soil pressure decreases, that induces the decrease of subsidence relative to the case of single pile and, as a consequence, the reducing of negative friction force. There exists experimental confirmation of this effect [1] and theoretical substantiation of its usage, the latter being founded on solving the equation of equilibrium of the HS and realized in TSN<sup>3</sup> 50-306-2005, sec. 6. Due to the named TSN, the vertical pressure in HS between reinforcing members of the strengthened basement can be evaluated by the formula:

$$\sigma_z = \frac{\gamma_{II} - \alpha_p c_{II}}{\alpha_p \xi \operatorname{tg} \varphi_{II}} \left( 1 - e^{-\alpha_p \xi z \operatorname{tg} \varphi_{II}} \right). \quad (1)$$

Here  $\alpha_p = p_{\Sigma}/S_{soil}$  is the ratio of the reinforcing member perimeter in field  $p_{\Sigma}$  to the area of inter-pile space in plan view  $S_{soil}$ ;  $\gamma_{II}$ ,  $c_{II}$ ,  $\varphi_{II}$  are the specific gravity, adhesion factor, and angle of internal friction of soil;  $\xi$  is the conjugate ratio (side-long pressure factor);  $z$  is the distance between a pit bottom and given cross-section of reinforcing member.

The formula (1) is the solution of the equation of equilibrium of the watered HS's layer of thickness  $dz$  in the case of a PF with infinite extension on rectangular grid. This equation is presented below; the solution (1) is obtained in supposition of no external exertion of pressure on the soil, when  $\sigma_z(z) = 0$  for  $z = 0$ , and supposition of the Coulomb's sliding of soil at the surface of reinforcing member, when distributed load of negative friction is determined by the expression:

$$\tau = c_{II} + \xi \sigma_z \operatorname{tg} \varphi_{II}. \quad (2)$$

The pressure determined by (1) is essentially less, as usual, then the pressure in non-reinforced thickness of soil, and with a short step of members it is possible to fulfill the inequality:  $\sigma_z < p_{sl}$  for all the depth of the HS ( $p_{sl}$  is the initial pressure of hydrocompaction subsidence). If this inequality is fulfilled, the authors of method conclude that negative friction is negligible, and the soil evokes the resistance over all lateral surface of reinforcing member [2].

Formula (1) can be restrictedly used for a soil in upper part of the HS, but it wouldn't be employed in over the all height of this thickness, not least because from the definite depth  $z$ , about on subsidence  $s_{sl}(z) = 5$  cm, the load at the lateral surface of pile changes the sign. The statement as itself about absence of negative friction, inspired through evaluation by formula (1), is contradictive, because just the negative friction enabled inference of this formula. Therefore, the analysis of reinforcing member's field due to TSN 50-306-2005 is not reliable. Nevertheless, the approach to collapse analysis based on solving the equation of HS equilibrium in inter-pile space with further estimation of pile bearing capability we would consider as correct. In the paper presented we suggest deformation hypotheses giving the way to develop such equation, have elaborated the solver for it, and founded the technique of finite-element simulation, enabled us to establish the necessary parameter of equation and check adequacy of results.

**Generalized Form of the Equation of HS State.** Let us consider an infinite field of rigid piles of length  $l$  on the square grid of pitch  $L_p$ . The PF is loaded to the settlement  $s_u$  with plane upper bound of soil, and next the watering and subsidence occur. Let us introduce the denotations for next quantities as functions of the depth  $z$ :

$\tau(z)$  is the friction force over the pile surface per unit of area (so called mantle friction), the sign “+” corresponds to downward direction;

$\sigma_z(z)$  is vertical pressure in inter-pile bulk of soil after watering;

$s_{sl}(z)$  is additional vertical displacement of soil after watering (the hydro-compaction subsidence).

We assume these functions being definite in segment  $[0, H_{sl}]$ , where  $H_{sl}$  is height of the HS.

Denote:  $\hat{\sigma}_z$  — operator of transformation  $\tau(z) \rightarrow \sigma_z(z)$ ;  $\hat{s}_{sl}$  — operator of transformation  $\sigma_z(z) \rightarrow s_{sl}(z)$ . The operators  $\hat{s}_{sl}$  and  $\hat{\sigma}_z$  are obtained below. Also, the following dependence is established for  $z \in [0, H_{sl}]$ :

$$\tau = \tau_0(z, \sigma_z, s_{sl}), \quad (3)$$

which enables to develop the equation of HS state:

$$\tau(z) = \tau_0(z, \hat{\sigma}_z \tau(z'), \hat{s}_{sl} \hat{\sigma}_z \tau(z')) \quad (4)$$

(in right-hand side  $\tau(z')$  is the function, exerted by corresponding operator). By solving this equation one can determine the stress-strain state (SSS) of the hydrocompactive stratum and tangent forces at the lateral surface of pile.

**Estimation of Mutual Displacement of Soil and Piles.** The construction of function (3) is based on estimations of mutual displacement of soil and pile's cross-section at the same depth. The displacement of soil is taken at the vertical in the middle between adjacent piles in a row. We denote the values of mutual displacement of soil and piles as:  $\Delta s(z)$  — before watering;  $\Delta s_{sl}(z)$  — after watering. The deformed planes of soil and the quantities  $\Delta s$ ,  $\Delta s_{sl}$  are depicted in Fig. 1 (the section A–A is constructed along a row of piles, see the plan in Fig. 2). The named quantities are shown in Fig. 1 for the case where there is no pile

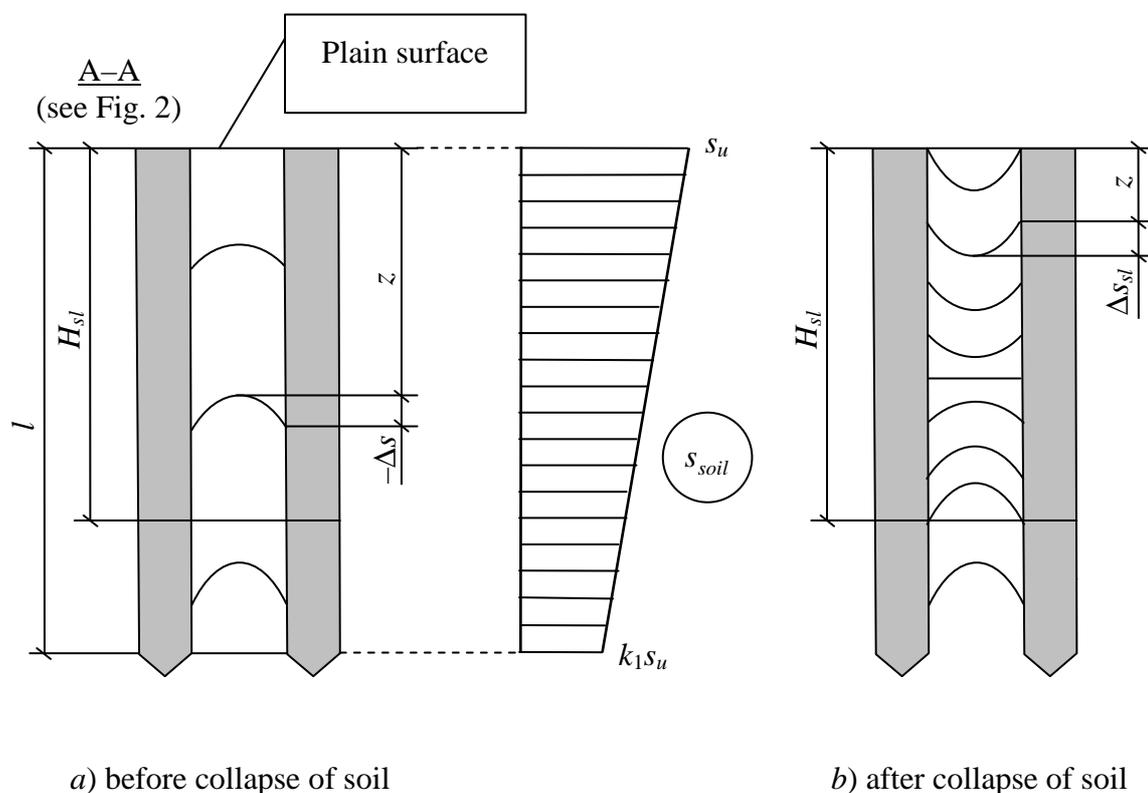


Fig. 1. Plots of mutual displacement of soil and piles

sliding in the soil stratum. We take the quantities  $\Delta s$ ,  $\Delta s_{sl}$  positive if the settlement of pile is less than displacement of soil at a given depth.

Hypothesis 1. The soil in stable state can be considered as elastic medium. The change in the soil structure due to watering evokes the change in its elastic properties.

Hypothesis 2. At the vertical in the middle between adjacent piles in a row the vertical displacement of the soil after foundation's settlement, but before watering and collapse, is defined by the formula:

$$s_{soil} = \frac{l - kz}{l} s_u, \quad (5)$$

where  $k = 1 - k_1$ ,  $k_1$  is the factor defining the bottom ordinate of the displacement diagram as  $k_1 s_u$ , see Fig. 1.

Let us establish additional displacement of the soil produced by watering. The change of the compression strain in a given horizontal section of the soil bulk we shall take for the change of the strain's elastic component caused by the pressure variation on watering plus the relative hydrocompaction subsidence:

$$\Delta \varepsilon_{sl} = \psi_e (\sigma_z - \sigma_{z1}) + \varepsilon_{sl}; \quad \psi_e \equiv \frac{1}{E_e} \frac{1 - \nu - 2\nu^2}{1 - \nu}. \quad (6)$$

Here  $\sigma_{z1}$  is vertical pressure before collapse;  $E_e$  is the soil deformation modulus;  $\nu$  is the Poisson's ratio;  $\psi_e$  — the soil compliance characteristic on tightened compression (i.e., compression with no transverse extension) [3];  $\varepsilon_{sl} = \varepsilon_{sl}(z, \sigma_z)$  is the relative hydrocompaction subsidence, also referred as the collapse strain, which is determined by tabulated function from experimental data.

Hypothesis 3. The change of the soil vertical deformation due to watering is defined by the relative hydrocompaction subsidence only.

Substantiation: The change of the vertical stresses  $\Delta \sigma_z = \sigma_z - \sigma_{z1}$  in formula (6) is the orders of magnitude no greater than the change of vertical stresses in unwatered soil after settlement of foundation, because both these quantities are defined by the friction forces on the piles, which are approximate-

ly similar (but, may be, of opposite signs). Thus, the component of elastic deformation in formula (6) also is the orders of magnitude not greater then the strain of unwatered soil within the bounds of piles  $\varepsilon_{soil} = -ds_{soil}/dz$  — even if we take the soil compliance  $\psi_e$  as the same in both cases. Having in mind that on collapsibility of type II the strain  $\varepsilon_{soil}$  is much lesser then the collapse strain after watering, we have already to expect that augend in formula (6) is much less in order of magnitude then addend. But in reality, the soil compliance in formula (6) is much less then compliance of soil being squeezed during foundation settlement. Really, the soil in HS get unloaded during watering, and, besides, the soil deformation modulus during relief is essentially greater then deformation modulus on the branch of initial loading — for example, in accordance to i.5.5 of SP 50-101-2004 we can take  $E_e = 5E$ . Thus, the component of elastic deformation in formula (6) is negligible in comparison to the collapse strain  $\varepsilon_{sl}$ .

Farther, in calculation of hydrocompaction subsidence we do the substitution:  $\varepsilon_{sl} \rightarrow k_{sl}\varepsilon_{sl}$ , where  $k_{sl}$  is the value over the range 1÷1.25 with meaning of the reliability coefficient (see SP 22.13330.2011 “SNIP 2.02.01-83 The Bases of Buildings and Constructions”, i.6.1.13). From expression (6), accounting all the said, we get additional displacement of the soil (the subsidence):

$$s_{sl} = \int_z^{H_{sl}} k_{sl}\varepsilon_{sl}(z, \sigma_z) dz + s_H. \quad (7)$$

Here  $s_H$  is the displacement of HS foot, which is conditioned by variation of the pressure acting on the footing plane during subsidence. Formula (7) defines the operator  $\hat{s}_{sl}$  in the set of continues functions  $\sigma_z(z)$ .

Hypothesis 4. It is allowable to take  $s_H = 0$  in hydrocompaction subsidence calculation.

Substantiation: The HS’s unloading after watering means that  $s_H < 0$ . The displacement  $s_H < 0$  diminishes the subsidence (7), i.e., reduces negative friction and increases resistance over the pile’s lateral surface outside of negative

friction domain. Therefore, the negligence of quantity  $s_H$  gives overdesign in bearing capability of a pile.

Denote  $s_1 = k_1 s_u$ . Before watering, the mutual displacement of soil and absolute rigid piles is obtained from expression (5) as follows:

$$\Delta s = s_{soil}(z) - s_u = -\frac{kz}{l} s_u = \frac{(k_1 s_u - s_u)z}{l} = (s_1 - s_u) \frac{z}{l}. \quad (8)$$

After watering we have the mutual displacement for  $z \leq H_{sl}$ :

$$\Delta s_{sl} = s_{sl} + \Delta s. \quad (9)$$

**The Analysis of Friction over Lateral Surface of a Pile.** The calculation formulas for function (3) determination we develop in 2 stages: evaluate the tangent distributed forces in assumption of elastic soil with given shear modulus  $G$ , and then correct them taking into account inelastic properties of a soil. We calculate the shear modulus in the form:

$$G = \frac{E_{ws}}{2(1 + \nu)},$$

where  $E_{ws}$  is the deformation modulus of completely water-saturated soil.

Let us cut off the soil ring of diameter  $L_p$  and height 1 m around a pile of round cross-section of diameter  $D$  at the wanted depth (Fig. 2). We assume simplistically the SSS of the ring as two-dimensional axis-symmetric, the possibility of sliding the soil along a pile is not accounted, the displacement of side surface of the ring relative to the pile we set equal to  $\Delta s_{sl}$ . This displacement is resulted by shear stresses in two-dimensional axis-symmetric problem. From equilibrium condition of any internal ring of radius  $r$  and height 1 m we derive as follows:

$$\tau(r)2\pi r = \tau_1 \pi D;$$

$$\tau(r) = \tau_1 \frac{D}{2r}; \quad \gamma = \frac{\tau(r)}{G}; \quad ds = \gamma dr,$$

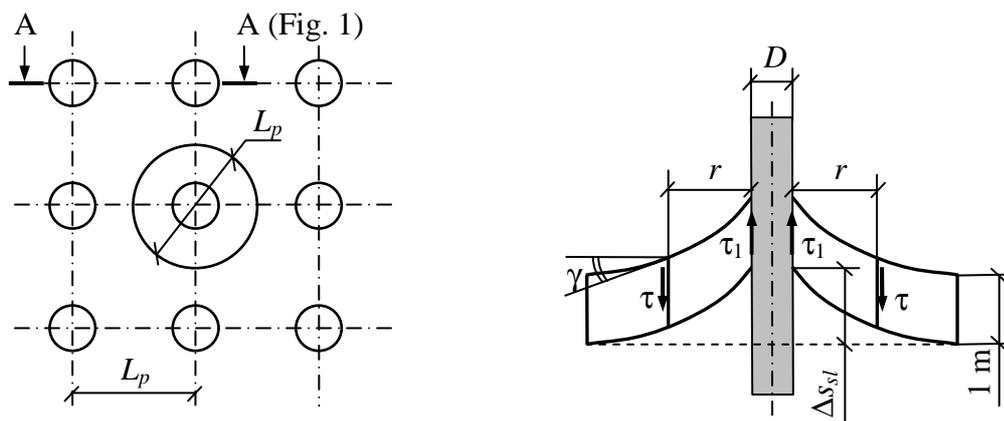


Fig. 2

where  $\tau_1$  is the sought friction loading over the pile surface;  $\gamma$  is the shear angle (Fig. 2);  $ds$  is the increment of vertical displacement of the ring along the distance  $dr$ . Thus it holds:

$$ds = \frac{\tau_1 D}{2rG} dr \Rightarrow \Delta s_{sl} = \int ds = \frac{\tau_1 D}{2G} \int_{D/2}^{L_p/2} \frac{dr}{r} = \frac{\tau_1 D}{2G} \ln \frac{L_p}{D} . \Rightarrow$$

$$\tau_1 = \frac{2G \Delta s_{sl}}{D \ln \frac{L_p}{D}} . \quad (10)$$

In the case of drilled pier the friction force over the pile surface is equal to the force of soil internal friction, which can't be greater then Coulomb's friction. For the latter we have:

$$\tau_{\max} = \xi \sigma_z \operatorname{tg} \varphi_I + c_I ,$$

where  $\xi = \frac{\nu}{1-\nu}$  is the sidelong pressure factor;  $\varphi_I, c_I$  are the soil strength characteristics for first limit state. For the mantle friction force (3) we obtain:

$$\tau_0(z, \sigma_z, s_{sl}) = \begin{cases} \tau_1(\Delta s_{sl}), & \text{if } |\tau_1| < \tau_{\max}; \\ \tau_{\max}, & \text{if } \tau_1 \geq \tau_{\max}; \\ -\tau_{\max}, & \text{if } \tau_1 \leq -\tau_{\max}. \end{cases} \quad (11)$$

Here  $\tau_1(\Delta s_{sl})$  is determined by (10); the positive surface force  $\tau_0$  corresponds to negative friction;  $z \leq H_{sl}$ .

In case of driven pile the mantle friction is established by Table 7.3 of SP 24.13330.2011. In this table the depth of a soil layer is in use instead of pressure on the pile surface, which defines the friction forces directly. To specify the layer (sublayer) depth due to this table, let us introduce the congruent level mark of pit bottom. The congruent level is defined as the bottom's level for such conventional pit *with no* PF, wherein under its bottom the vertical pressure in given layer (sublayer) is the same as in real pit *having* PF with design settlement in the state of watering and subsidence. For any soil sublayer used in analysis, congruent level is of its own value. Conventional pit with bottom of congruent level is utilized to determine the sublayer's depth being used in Table 7.3 of SP 24.13330.2011. The need of the congruent level calculation lies in the fact that data of above mentioned table are established by experiment under conditions where evaluation of the soil pressure is tolerated with no account of soil-pile interaction, i.e., by the standard technique from SP 22.13330.2011, formula (5.23). Note, that after having calculated congruent level, one has to converse it into conditional level of the pit bottom, q.v. Remark 2 for Table 7.2 from SP 24.13330.2011.

In the case of driven pile the formula (11) remains in effect, but now the quantity  $\tau_{\max}$  is the tabulated design strength of a soil.

### **The Analysis of Pressure $\sigma_z$ in the Watered Hydrocompactive Soil.**

Let us obtain the operator  $\hat{\sigma}_z$ . Within the boundaries of a soil layer of thickness  $dz$  the stress value augments by  $d\sigma_z$  as a result of the soil weight exertion, on one hand, and the mantle friction forces on the pile surface, counteracting to this augmentation (Fig. 3). The equilibrium equation for vertical forces in the layer has the form:

$$S_{soil} \cdot d\sigma_z = \gamma(z) \cdot dz \cdot S_{soil} - dF; \quad dF \equiv p_{\Sigma} dz \cdot \tau(z). \quad (12)$$

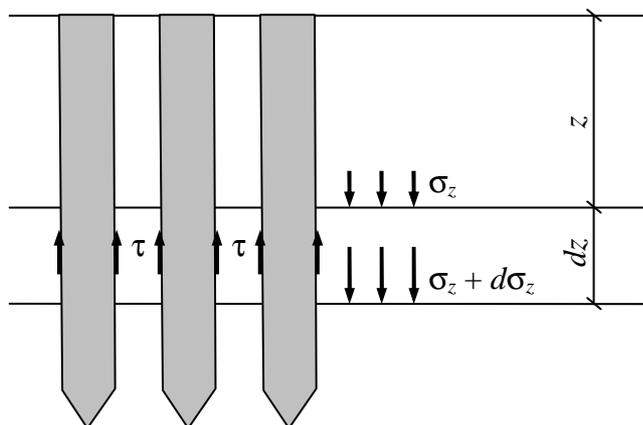


Fig. 3. Diagram of load action in the foundation base layer of thickness  $dz$

Here  $F$  is the total force of negative mantle friction above the depth  $z$ ;  $p_\Sigma$  and  $S_{soil}$  are entered before in formula (1);  $\gamma(z)$  is the normative specific weight of soil<sup>1</sup>. After division of both parts in equation (12) by  $S_{soil}$ , we come to equation:

$$d\sigma_z = \gamma dz - \alpha_p dz \tau(z), \quad (13)$$

whereas for  $z = 0$  is  $\sigma_z = 0$ . Respectively:

$$\sigma_z(z) = \hat{\sigma}_z \tau(z') = \int_0^z (\gamma - \alpha_p \tau(z')) dz'. \quad (14)$$

The equation (13) with substitution (2) has analytic solution (1).

The equation of the HS state (4) is defined by obtained relationships. In part 2 of the paper we substantiate an algorithm of its solving and technique of FEM-simulation of the hydrocompactive stratum's SSS. The latter enables to determine the equation's parameter  $s_1$ , that is the settlement of soil at the foot of conventional foundation, see formula (8). Besides, the analysis is presented for computational results of the safe bearing load on a pile by new method in comparison to standard technique from SP 24.13330.2011.

<sup>1</sup> Stability of the soil specific weight makes possible to evade the usage of its confidence limits in first limit state.

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<sup>1</sup> SP — Set of Rules (Svod Pravil).

<sup>2</sup> SNIP — Building Code and Regulations (Stoitelnie Normi i Pravila).

<sup>3</sup> TSN — Regional Building Code (Territorialjnie Stoitelnie Normi).